Tidal radii of globular clusters and the mass of the Milky Way

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ABSTRACT

Tidal radii of remote globular clusters ($R_{GC} \geqslant 35$ kpc) are used to provide constraints of the mass profile of the Milky Way galaxy that are independent of kinematic data. The available data are consistent with the profile of an isothermal sphere with circular velocity $V_c = 220 \pm 40$ km/s in the radial range 35 kpc $\leqslant R_{GC} \leqslant 100$ kpc, in good agreement with all recent estimates. The more robust constraint at large distances from the galactic center is provided by NGC 2419, yelding an enclosed mass of $1.3^{+2.9}_{-1.0} \times 10^{12}$ $M_\odot$ at $R_{GC} \simeq 90$ kpc.

Key words: Galaxy: fundamental parameters - globular clusters: general - dark matter

1 INTRODUCTION

One of the most straightforward examples of the great difficulties associated with the measure of basic physical quantities on the astrophysical scales is provided by the quest for the mass of the Milky Way. Even applying the most refined analysis and using the whole wealth of available data, realistic uncertainties affecting single estimates of such a fundamental parameter tipycally amount to $\sim 100 - 300$ % (see, e.g. Wilkinson & Evans 1999, for a state-of-the-art analysis). The whole problem was recently reviewed and critically discussed by Zaritsky (1999, hereafter Z99). This author notes that the concept of total mass of the Milky Way is somehow ill-defined since we ignore the actual extent of the Dark Matter (DM) halo of the Galaxy. Hence, it is much safer to refer mass estimates to the *enclosed mass* within the galactocentric distance (R_{GC}) sampled by the mass-tracer under consideration. This approach allows a sensible comparison between different estimates, since consistency requires that all estimates shall be in agreement with a unique mass profile $[M(R_{GC})]$ over the whole range of galactocentric distances that can be probed. Z99 uses the *isothermal sphere* as a reference model to perform such a comparison and concludes that all the available estimates, covering different ranges in R_{GC} and using different tracers (from the HI rotation curve to the outermost Galactic satellites), are consistent with the mass profile of an isothermal sphere with rotational velocity $V_c \sim 180$ km/s. Note that in this context the isothermal sphere is (obviously) *not* intended as a realistic model for the Galactic DM halo, but just as a suitable mass profile to provide comparison and cross-validation of the various estimates. Hence, despite the large uncertainties affecting the single mass estimates, a remarkable overall consistency is apparent. If interpreted in the classical Newtonian framework, the mass of the Galaxy (M_G) is observed to grow approximately as $M_G \propto R_{GC}$ and the enclosed mass within $R_{GC} \simeq 300$ kpc is of order $M_G \sim 2 \times 10^{12}$ $M_\odot$ (Z99).

All the estimates considered by Z99 (as well as the more recent ones in the post-'99 literature) rely on the kinematics of the adopted tracers (rotational velocity curve, escape velocity of local stars, motion of satellites, either stars, globular clusters or dwarf galaxies). Therefore, an estimate of $M_G(R_{GC})$ not based on kinematical data would provide a further important consistency check of our ideas on the mass profile of the Galaxy. Such kind of probe may be provided by tidal radii of globular clusters (GC) (von Hoerner 1956; King 1962). Theory predicts that the Galactic tidal field fixes the cut-off in the density profile of GCs, the position of the cut-off depending on the cluster distance and on the ratio between the mass of the Galaxy and the mass of the cluster (m_c). Hence, having m_c and R_{GC} from observations, an estimate of M_G may be obtained, though affected by large uncertainties. The approach has been attempted in the past by Wakamatsu (1981, hereafter W81) and by Innanen, Harris & Webbink (1983, hereafter IHW). W81 used globular clusters and dwarf spheroidal galaxies as tracers, and obtained $M_G(78\ kpc) = 2.0^{+1.3}_{-0.7} \times 10^{11}$ $M_\odot$. The result is in marginal disagreement with the conclusions by Z99 but is probably affected by the large uncertainties in the distance and tidal radii of dwarf spheroidals (for the same reasons these galaxies are not considered in the present analysis). IHW used only GCs and found $M_G(44\ kpc) = 8.9 \pm 2.6 \times 10^{11}$ $M_\odot$, consistent, within the uncertainties, with the Z99 results.

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There are cogent reasons to try to repeat the experiment twenty years later. The quality of the observational material is greatly improved, mainly thanks to the extensive compilation of surface brightness profiles by Trager, King & Djorgovski (1995), and theoretical advancements put the problem in a new light (Oh, Lin & Aarseth 1992, 1995; Meziane & Colin 1996; Brosche, Odenkirchen & Geffert 1999). Moreover the impressive growth of computing facilities allow an accurate analysis of the uncertainties (a particularly critical point for this kind of application, see IHW for a discussion) by means of extensive Montecarlo simulations.

In this paper I review the use of tidal radii of globular clusters in the light of the more recent theoretical and observational result and I check if the mass estimates from this technique are consistent with "kinematical" estimates, dealing in particular with the mass profile at large R_{GC}, e.g. the most interesting range (Sect. 2). The main conclusions and future prospects are summarized in Sect. 3.

2   TIDAL RADII AS MASS PROBES

A general formula for the tidal radius (r_t) of a globular cluster of mass m_c orbiting around a galaxy of mass $M_G(R)$ is (King 1962; Oh, Lin & Aarseth 1992, IHW, W99):

$$r_t = k \left[\frac{m_c}{f(e)M_G(R_p)} \right]^{\frac{1}{3}} R_p \qquad (1)$$

where R_p is the peri-galactic distance, $f(e)$ is a function depending on the form of the galactic potential and on the eccentricity of the cluster orbit e and k is a factor, firstly introduced by Keenan (1981a,b) to account for the elongation of the limiting tidal surface along the line between the cluster center and the galactic center. According to this author, IHW and W99 assume $k = \frac{2}{3}$. From now on we drop, for brevity, the explicit indication of the dependence of mass on distance: in any case M_G shall be intended as the mass enclosed with a given galactocentric distance R (the corresponding index is also dropped, from now on R means R_{GC}).

The dependence on e was introduced by King (1962) by reformulating the equation for the instantaneous tidal radius of von Hoerner (1956) in terms of the perigalactic distance (instead of the instantaneous distance R). This choice was motivated by the fact that the typical orbital period (P) of globulars was expected to be much shorter than the internal relaxation time (tipically quantified by the half-mass relaxation time t_{rh}). In this case two-body relaxation is unable to keep the external structure of the cluster at pace with the changing galactic tidal field. In other words, the tidal force at perigalacticon truncates the cluster at the corresponding radius and the internal relaxation is too slow to restore a larger limiting radius before the next perigalactic passage (see also Oh, Lin & Aarseth 1995).

However recent comparisons between predicted and observed tidal radii that included all the information on cluster orbits (as derived from measured proper motions Meziane & Colin 1996; Odenkirchen et al. 1997; Brosche, Odenkirchen & Geffert 1999) showed that observed tidal radii are larger than one would expect if they were fixed at the perigalactic point. Meziane & Colin (1996) suggest that the actual tidal radius depends on the orbital
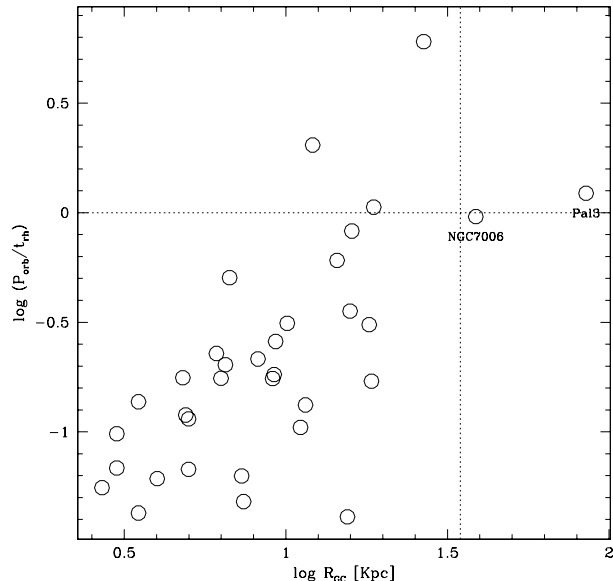

Figure 1. The logarithm of the ratio between the orbital period (P) and the half-mass relaxation time is plotted versus the galactocentric distance for all the Galactic globulars for which orbital parameter are available in Dinescu, Girard & van Altena (1999) or Dinescu et al. (2001). The horizontal line marks the threshold $P/t_{rh} = 1$, the vertical line marks $R_{GC} = 35$ Kpc. The names of the clusters with $R_{GC} > 35$ Kpc are also reported.

phase, while Brosche, Odenkirchen & Geffert (1999) argue that a suitable average along the orbit defines a much more proper tidal radius with respect to the perigalactic value.

Oh, Lin & Aarseth (1992) and Oh, Lin & Aarseth (1995) studied in detail the properties of tidal radii of globular clusters by means of N-body simulations. They found that while the tidal limit of clusters with $P/t_{rh} << 1$ is effectively set at perigalacticon, in clusters with $P/t_{rh} \sim 1$ internal relaxation effects are able to repopulate their external regions after the perigalacticon passage and their tidal radius may be comparable to the value expected at *apogalacticon*.

In Fig. 1 the logarithm of the $Q_t = P/t_{rh}$ ratio is plotted versus the galactocentric distance for all the clusters for which orbital parameters from measured proper motions are available (from the computations by Dinescu, Girard & van Altena 1999; Dinescu et al. 2001). The relaxation times are from the 2003 version of the Harris (1996) catalogue, which is the source of all the globular cluster data used in this study if not otherwise stated. There is a clear correlation between Q_t and R suggesting that beyond $R \sim 35$ kpc clusters are very likely to have $Q_t \gtrsim 1$, i.e., according to the results by Oh, Lin & Aarseth (1995), their r_t does not reflect the tidal force at perigalacticon.

On the basis of all the above considerations I make the assumption that *the observed tidal radii of distant globular clusters (e.g., those with $R \geqslant 35$ kpc) are probes of the Galactic tidal force at their present position.* In other words I take the galactocentric distance $R_p \leqslant R \leqslant R_a$ as a reasonable approximation of the effective orbital radius at which the tidal radius of clusters with $Q_t \gtrsim 1$

is fixed (Meziane & Colin 1996; Meylan & Heggie 1997; Odenkirchen et al. 1997; Brosche, Odenkirchen & Geffert 1999; Oh, Lin & Aarseth 1995). The assumption limits the range that can be probed to the outer halo of the Galaxy. On the other hand this is the most interesting range since the contribution of barionic matter to the mass budget is expected to be negligible in this region. Furthermore, clusters orbiting in such remote regions are less likely to have their structure changed by close encounters with the galactic disk, bulge or with large molecular clouds, and are therefore more reliable probes of the Galactic tidal field (see Meylan & Heggie 1997, for an extensive review of the mechanisms affecting the structure of GCs).

Following the above assumption, the following form of the theoretical tidal radius is adopted:

$$r_t = k \left[\frac{m_c}{2M_G} \right]^{\frac{1}{3}} R \qquad (2)$$

e.g, Eq. 1 with $f(e) = 2$ and R instead of R_p. In the context of a logarithmic potential this is the exact formula for the tidal radius of a cluster on a circular orbit (King 1962). Solving with respect to M_G:

$$M_G = \frac{1}{2} k^3 m_c \left(\frac{R}{r_t} \right)^3 \ . \qquad (3)$$

W81, IHW and Oh, Lin & Aarseth (1995) derived $f(e)$ for eccentric orbits in a logarithmic potential. It is important to note that the adoption of $f(e) = f_{log}(e)$, instead of the instantaneous value $f(e) = f_{log}(e = 0)$ adopted here, has a modest impact on the final M_G estimates. The adoption of the $f_{log}(e)$ by Oh, Lin & Aarseth (1995) changes the final M_G estimate by a factor $\leqslant 2$ for $e \leqslant 0.825$ and by a factor $\leqslant 4$ for $e \leqslant 0.935$ with respect to the $f(e) = 2$ case adopted here. In any case the derived M_G is larger than what obtained with Eq. 3. In analogy with the analysis that will be described in Sect. 2.2 I performed an extensive set of Montecarlo experiments using the $f_{log}(e)$ version of Eq. 1 and exploring the whole range of possible orbital eccentricities. The resulting mass (and mass profile) estimates are fully consistent with the results obtained from Eq. 3. Hence, the approximations involved in the adopted approach appear fully adequate for the present purpose (see also Ibata et al. 2001).

2.1 The observational side

Tidal radii of GCs are not, in general, directely observable quantities. Fitting King models (King 1962) to surface brightess profile of globular clusters one obtains an estimate of the core radius (r_c) and of the concentration parameter $C = \log \left(\frac{r_t}{r_c} \right)$; r_t is derived from these fitted parameters. Hence, the best estimates of r_t may be obtained from bright clusters (e.g., providing high signal-to-noise data for the fit) whose surface brightness profile is reliably measured over the largest possible radial range (thus limiting at a minimum the extrapolation to the actual tidal limit).

Unfortunately the range $R_{GC} > 35$ kpc is mainly populated by sparse, low luminosity clusters whose tidal radii and integrated magnitudes are quite uncertain. The suitable galactic clusters in the relevant range of galactocentric distances are: Pal 15, NGC 7006, Pyxis, Pal 14, NGC 2419, Eridanus, Pal 3, Pal 4 and AM-1. Pal 15 and Pyxis have

been excluded since they are affected by significant amount of interstellar extinction ($A_V > 0.6$ mag, that implies a larger uncertainty in the estimate of m_c). Note however that the inclusion of these clusters does not change in any way the final results presented below. Of the remaining clusters the best suited for the present analysis are NGC 7006 and NGC 2419, two bright and well studied clusters. In particular NGC 2419 is the 4-th most luminous cluster of the whole Galaxy ($M_V = -9.58$) and its surface brightness profile have been reliably measured out to the 85 % of the deduced tidal radius, i.e. the level of extrapolation is quite modest. Finally NGC 2419 is the only cluster of the considered set for which a direct estimate of the mass-to-light ratio (M/L, a fundamental ingredient to derive m_c) is available ($M/L_V = 1.2$, from Pryor & Meylan 1993).

In conclusion, NGC 7006 and NGC 2419 provide by far the most robust and less uncertain mass probes, the remaining clusters are retained just for consistency check.

2.1.1 Montecarlo simulations

M_G is estimated from each cluster using Eq. 3. To deal with uncertainties I obtained 10000 independent M_G estimates for each cluster by extracting at random (from suitable distributions described below) the following quantities: the observed V-band distance modulus μ_V, the apparent integrated V magnitude V_t, the tidal radius in arcmin r_t, the V-band mass-to-light ratio in solar units M/L and Keenan's k factor (see below). The color excess $[E(B-V)]$ is kept fixed, since it is small (less than ~ 0.1 mag) in all of the considered cases. For each set of extracted parameter the following items are computed: the distance from the Sun and the Galactic Center from μ_V, $E(B-V)$ and the galactic coordinates, the linear tidal radius from its angular value and distance, the mass of the cluster from V_t, μ_V and M/L, and finally, from R and r_t in kpc and m_c in solar masses, the Galactic mass enclosed within R. All the distributions are chosen to (conservatively) cover the whole range that is compatible with the adopted uncertainties of each parameter, hence the final 10000 M_G estimates cover the whole range allowed by taking into account *all the possible sources of error*. Finally the median of the 10000 estimates is computed as well as the range in M_G including the 90 % of the derived M_G estimates.

2.1.2 Uncertainties on input parameters.

To have a closer look to the details of the simulations, the assumptions on input parameters are shortly described below.

- k factor. Keenan's factor ($k = \frac{2}{3}$) provides an average correction for the non-spherical shape of the limiting tidal surface. To account for the possible cluster-to-cluster variation of this parameter, k is extracted from a uniform distribution in the range $0.5 \leqslant k \leqslant 1.0$ (see also Heggie & Ramamani 1995).
- μ_V. To account for both the measurement errors and the uncertainties still affecting the distance scale of globular clusters (see Cacciari 1999, and references therein) μ_V values were extracted from a gaussian distribution with mean equal to the μ_V listed by Harris (1996) and standard deviation

$\sigma_\mu = 0.1$ mag for NGC 2419 and NGC 7006 and $\sigma_\mu = 0.2$ mag for the remaining, less extensively studied, clusters.

• V_t values were extracted from a gaussian distribution with mean equal to the V_t listed by Harris (1996) and standard deviation $\sigma_V = 0.1$ mag for NGC 2419 and NGC 7006 and $\sigma_V = 0.2$ mag for the remaining clusters. Note that the measure of this parameter is particularly critical for low-density and low-brightness clusters, like the majority of those considered here.

• M/L. The observed Fundamental Plane of GCs (Djorgovski 1995; Bellazzini 1998) implies that the M/L ratio of globular clusters is constant to within a factor of a few, approximately compatible with the measurement errors. Pryor & Meylan (1993) find $0.5 \gtrsim M/L \gtrsim 4$ in good agreement with Mandushev, Spassova & Staneva (1991). Here the M/L values are extracted from a gaussian distribution with mean $M/L = 1.2$ (e.g., the estimated M/L of NGC 2419 and the average M/L derived by Mandushev, Spassova & Staneva 1991) and $\sigma_{M/L} = 0.4$, with the further constraint $M/L \geqslant 0.5$. This assumption ensures that the range of observed M/L of GCs is fully explored.

• r_t. I searched the literature to find estimates of r_t for the considered clusters that may supersede those reported by Harris (1996), all drawn from Trager, King & Djorgovski (1995). The only (partially) successful case was Pal 14, for which I adopt the estimate by Harris & van den Bergh (1984) whose density profile is marginally more extended with respect to that by Trager, King & Djorgovski (1995). The adopted r_t values are extracted from a gaussian distribution with the mean equal to the listed values and with standard deviation in the range $\sigma_r = 0.1r_t - 0.3r_t$, depending on the quality and extension of the available surface brightness profile. In particular $\sigma_r = 0.1r_t$ for NGC 2419, $\sigma_r = 0.2r_t$ for NGC 7006 and $\sigma_r = 0.3r_t$ for the remaining clusters.

The Referee correctly pointed out that the estimates of tidal radii may be also plagued by systematics. For example the adoption of different models (e.g. Wilson 1975) to fit the surface brightness profiles of globulars may lead to obtain significantly larger limiting radii than what estimated with King's models (see McLaughlin & Meylan 2003, for an application). Moreover, some theoretical studies also suggest that realistic models of globular clusters in the Galactic tidal field may be slightly more spatially extended than King's models (see, e.g. Kashlinsky 1988; Heggie & Ramamani 1995). To explore the effect of systematics that may change the observationally estimated r_t values up to a factor ~ 2 I repeated the analysis described above for NGC 2419 ($rt = 8.74'$, according to Trager, King & Djorgovski 1995) assuming $\sigma_r = 0.5r_t$ instead of $\sigma_r = 0.1r_t$. To preserve compatibility with the observed profile, which reaches $r \simeq 7.5'$, I forced the randomly extracted tidal radii to the range $r_t > 6'$. Hence the final r_t of the Montecarlo simulation are in the range $6' < r_t \gtrsim 20'$. Note that at $r \geqslant 6'$ the observed profile is rapidly falling and the surface brightness is $\Sigma_V > 28$ mag/arcsec^2, thus it is quite unlikely that the actual limiting radius is much larger than $10'$. With these new assumptions the typical uncertainty on the final M_G estimates grows from $\sim 75\%$ (for the case $\sigma_r = 0.1r_t$) to $\sim 110\%$ while the derived me-

dian M_G is practically unchanged. Therefore the inclusion of these possible systematic errors in the uncertainty budged doesn't seriously affect the main conclusions of this paper (see Sect. 2.2), at least for what concern the cluster that provide the most interesting constraint on M_G, e.g. NGC 2419.

2.2 The mass of the Galaxy within $35 < R_{GC} < 100$ kpc.

Fig. 2 reports the median M_G and R of the 10000 simulation carried on for each considered cluster (filled circles). The error bars enclose 90 % of the derived estimates (± 45 %). The most reliable points (NGC 7006 and NGC 2419) are indicated by larger symbols.

Other mass estimates, more recent than those considered by Z99, are also reported (see legend). The only pre-99 estimate reported is that by Kochanek (1996), who provides the more extensive treatment of the problem at that epoch. The filled triangle with an arrow, labeled L-B99 in the legend, provides a sensible upper limit to the total mass of the Galaxy since it is the estimate of the mass of the Local Group obtained by Lynden-Bell (1999) using the Local Group timing technique. The reported error bars have etherogeneous meanings ($1 - \sigma$ errors, 90 % c.l., etc., as provided by the authors) and hence they are not directly comparable. The mass profiles of isothermal spheres with $V_c = 180, 220, 260$ km/s are also plotted for reference. They are intended to allow the comparison of the estimates at large R with constraints provided by the rotational velocity of the HI disk at $R < 20$ kpc, taking into account the whole range of possible uncertainty on the Galactic V_c (see Fitch & Tremaine 1991; Pedreros, Anguita & Maza 2002, Z99, and references therein).

From the inspection of Fig 2 the following main conclusions can be drawn:

(i) All the reported estimates based on kinematical data are in agreement, within the uncertainties, with the reported mass profiles and with the results summarized by Z99.

(ii) All the estimates based on the tidal radii of globular clusters are consistent with the reported mass profile, within the (large) uncertainties.

(iii) The clusters that provide the most reliable constraint with the adopted technique (e.g., NGC 7006 and NGC 2419) give estimates of the enclosed Galactic mass in *excellent* agreement with the reported profiles and with all other estimates. In particular, from NGC 7006 I obtain $M_G(39$ kpc$) = 0.38^{+1.3}_{-0.1} \times 10^{12}\ M_\odot$, and from NGC 2419, $M_G(92$ kpc$) = 1.3^{+2.9}_{-1.0} \times 10^{12}\ M_\odot$.

Hence, the present application of the tidal radii technique provide an independent validation of the *standard* framework for the mass of the Milky Way as it emerges from the analysis by Z99 and from more recent studies. There is a quite remarkable general agreement among all the considered estimates, indicating that (a) the mass of the Galaxy appears to grow with galactocentric distance at least up to $R \gtrsim 100$ Kpc and (b) the total mass of the Galaxy is larger than $\sim 2 \times 10^{12}\ M_\odot$. This implies that the V-band mass to light ratio of the Milky Way spans a range $20 \gtrsim M/L \gtrsim 100$ in the radial range 30 kpc $\gtrsim R \gtrsim 200$ kpc (see Fig. 3 by Z99). Hence, in the classical Newtonian theory, all the available observational constraints consistently point to the con-
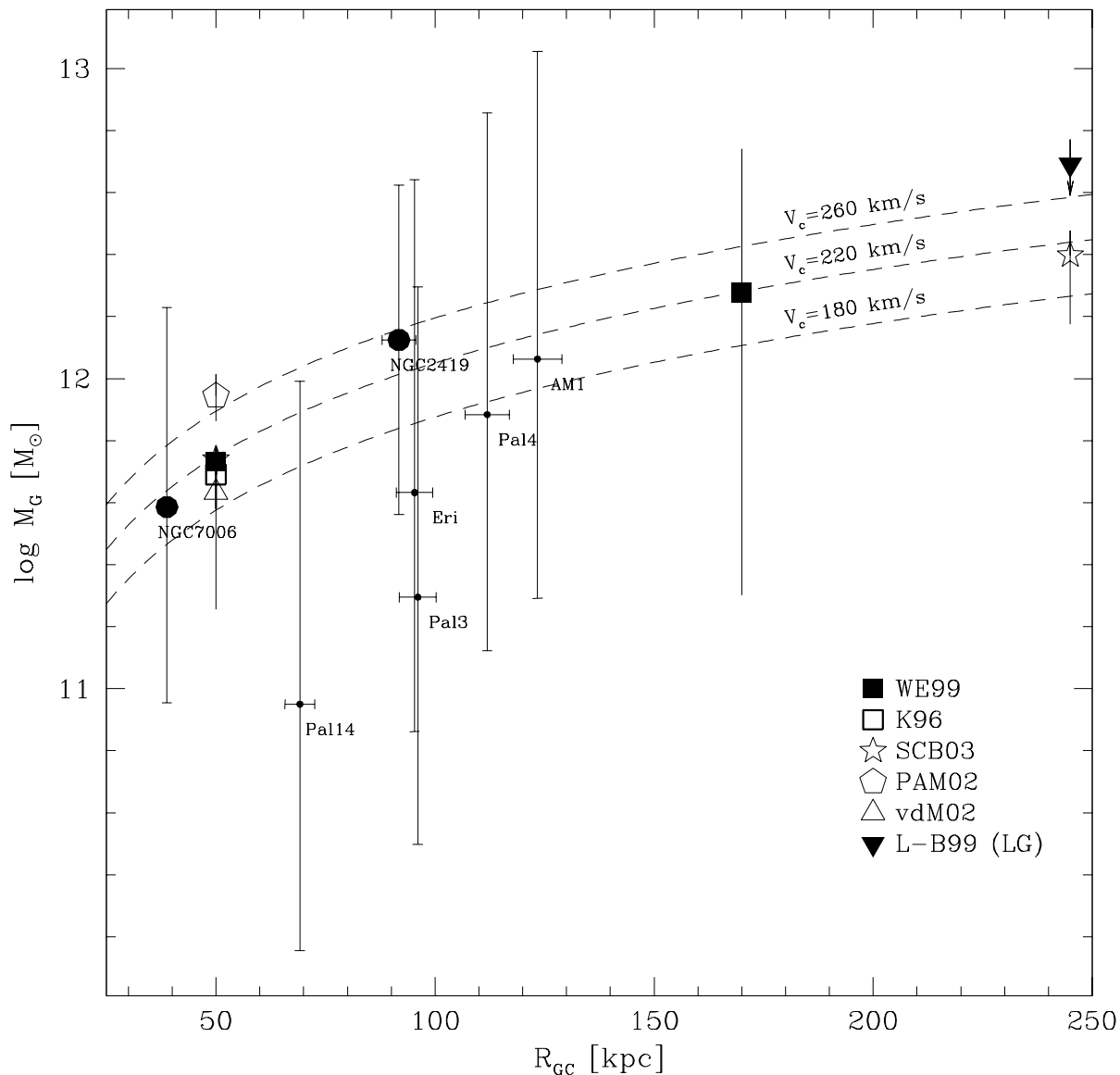
Figure 2. The Mass profile of the Milky Way halo (for $R_{GC} > 35$ kpc) from various recent estimates based on kinematical data (see legend in the lower left corner) and from the tidal radii of the selected globular clusters (filled circles). Large filled circles are the best observed globulars, providing (by far) the stronger constraint on M_G with respect to the other clusters. The error bars for globulars enclose the 90 % c.l. range. The mass profiles of isothermal spheres with $V_c = 180, 220, 260$ km/s are also reported for reference. Acronyms: WE99 = Wilkinson & Evans (1999); K96 = Kochanek (1996); SCB03 = Sakamoto, Chiba & Beers (2003); PAM02 = Pedreros, Anguita & Maza (2002); vdM03 = van der Marel et al. (2002); L-B99 = Lynden-Bell (1999), mass of the Local Group. The arrow indicates that the last one is an upper limit for the total mass of the Galaxy.

clusion that the Milky Way is surrounded by a huge Dark Matter halo.

3 CONCLUSIONS

The tidal radii of galactic globular clusters have been used to obtain estimates of the enclosed mass of the Galaxy in the range of galactocentric distances 35 kpc $\leqslant$ R_{GC} $\leqslant$ 100 kpc, under the assumption that, for

such remote clusters, R_{GC} is a reasonable approximation of the orbital radius at which their tidal limit is imposed (Meziane & Colin 1996; Odenkirchen et al. 1997; Brosche, Odenkirchen & Geffert 1999; Oh, Lin & Aarseth 1995). The adopted technique provides an estimate of the enclosed galactic mass that is independent of kinematical data that, on the other hand, provides the basis of all other existing estimates (see Sect. 1). Therefore, while the associated uncertainties are still quite large, the present appli-

cation provide (at least) an interesting consistency check of the existing kinematical estimates of M_G.

The estimates obtained from tidal radii are fully consistent with results from other authors and other methods. All the available constraints are consistent with the mass profile of an isothermal sphere with $V_c = 220 \pm 40$ km/s. Hence, the present analysis provides independent support to the fact that the mass of the Galaxy grows with R out to large distances from the Galactic Center and that the mass enclosed within $R \simeq 90$ kpc is $M_G \simeq 10^{12}$ $M_\odot$.

The present analysis indicates that tidal radii of remote globular clusters may be better probes of the galactic potential than previously believed (IHW). A detailed and comprehensive theoretical analysis it is now at hand with realistic N-body simulations (e.g., with GRAPE, Makino 1996) and can provide much sounder and solid basis to the technique. At the same time, wide-field cameras mounted on large telescopes may provide the opportunity to obtain more extended density profiles, based on star counts, hence reducing the observational uncertainty. These advancements may ultimately lead to a fully reliable additional technique to probe the mass profile of our Galaxy, nicely independent of and complementary to the usual kinematical methods.

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